Base Station-Assisted Cooperative Network Coding for Cellular Systems with Link Constraints

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Abstract—In this work, we consider a novel distributed data storage/caching scenario in a cellular network, where multiple nodes may fail/depart at the same time. In order to maintain the target reliability, we allow cooperative regeneration of lost nodes with the help of base stations allocated in a set of hierarchical layers. Due to this layered structure, a symbol download from each base station has a different cost, while the link capacities connecting the nodes of the cellular system and the base stations are also limited. In this general case, we present the fundamental trade-off between repair bandwidth cost and the storage space per node. Particularly interesting operating points are the minimum storage as well as bandwidth cost points in this trade-off curve. We provide closed-form expressions for the corresponding bandwidth (cost) and storage space per node for these operating points. Finally, we provide an explicit optimal code construction for the minimum storage regeneration point for a special set of system parameters.

I. INTRODUCTION

Scalability and reliability of distributed data storage and caching are quite crucial for next generation cellular system design [1]. In such settings, nodes that store information may fail or leave the cell permanently. For fault tolerance, the data is usually cached in erasure coded form in the cell. Moreover, in order to maintain the target reliability (or service quality), we need to regenerate the unavailable content within predetermined time intervals. A data of size $F$ symbols is chopped into $k$ fragments, erasure–encoded into $n$ fragments and finally distributed across different $n$ mobile nodes of the local cell where each node stores $\alpha$ symbols. A user can reconstruct the data by downloading all symbols that reside in any $k$ local nodes. In the case of a failure or node departure, either an instantaneous repair process regenerate lost node contents or a lazy repair may be adapted in which no action take place for a time duration before initiating a cooperative regeneration, both by downloading $\gamma$ symbols from other nodes. Dimakis et al. [2] considered a single node failure and formulated the fundamental trade-off between the data storage per node $(\alpha)$ versus the required bandwidth $(\gamma)$. On the other hand, the cooperative regeneration is shown to reduce the required bandwidth dramatically [3], [4]. In a lazy cooperative repair, there are two phases. Supposing that there are $t$ failed/departed nodes at the time of regeneration, empty nodes (newcomers) are regenerated by first contacting $d \leq n - t$ other live nodes to download $\beta \leq \alpha$ units of data to resurrect the lost data content (a total bandwidth of $\gamma = d/\beta$). In the second phase, after each newcomer processes the downloaded data, the $t$ newcomers exchange data among themselves, each by downloading $\beta'$ units of data from the rest of the other $t - 1$ newcomers.

In this paper, we consider a layered storage architecture, supported with multiple base stations (BSs) which cache the stored content in the nodes of the local cellular network. In particular, we formally introduce two novel concepts, namely bandwidth cost and link capacity in addition to cooperation. More specifically, we constrain the maximum link capacity limit to be $b_l/\beta$ to be downloaded from the BS at the $l$-th layer. The bandwidth cost $(\gamma_c)$ defines the price we pay per downloaded symbols from each BS.

We realize that despite there are similarities to previous rack-aware [5], [6] and clustered storage configurations [7] in which intra–rack/cluster and cross-rack/cluster bandwidths are introduced and tied together with a cost-like factor $\epsilon$, the essence of the problem is more general in our treatment. More specifically, $\epsilon$ is introduced as a ratio of inter (cross) cluster bandwidth $\beta_c$ and intra cluster repair bandwidth $\beta_l$ per node [7]. First and foremost, we defined the cost notion more formally and then we assumed capacity-constrained communication channels. A later work [8] addressed the cooperative scenario for rack-aware regeneration in a limited context where cooperation is allowed only among nodes of different racks. Moreover, no BS or a back-up server is utilized in such a setting. Alternatively, the repair process can be handled in a centralized fashion to further reduce the bandwidth requirements. In [9], repairing multiple nodes in a centralized strategy is studied for the regenerating codes. Despite this strategy reduces the repair bandwidth in general, when the number of losses is less than $k$ and more than one, it becomes impossible to achieve the minimum bandwidth for linear exact repair codes. Moreover such centralization may be the source of single point of failure and therefore constitutes a vulnerability in a cellular setting. On the other hand, Calis et al. [10] considered a hierarchical storage with backup nodes in order to improve the overall communication cost. Furthermore, they compared distributed and centralized repair strategies in their recent study [11]. However these studies did not take into account the cooperation which is quite instrumental in a local network to help regeneration process. Our optimization formulation as well as the final expressions can be verified to be extensions of the previous works. However consideration of BSs as helper nodes changes the problem dramatically.
and calls for distinct code constructions that would help us with cellular networks in which energy consumption and file popularity are important [12].

In Section II, we provide our motivation with a coding example in which BS is used, followed by the problem statement. In Section III, we provide our main result for the trade-off between the bandwidth cost and node storage. We explore two important operating points in the trade-off curve; namely BS-assisted minimum storage and minimum bandwidth cost cooperative regeneration (namely BS-MSCR and BS-MBCCR, respectively). In Section IV, we provide a code construction for BS-MSCR operating point with \( d = k \). Finally, we conclude our paper in Section V.

II. MOTIVATION AND SYSTEM MODEL

A. Motivating Example

Let us consider a recurring example in the literature whereby out of four storage nodes, two nodes fail and we attempt cooperative regeneration of data in these nodes. In that setting, each node stores two packets where each packet is assumed to be 1MB (\( F = 4 \) MB). The first node stores \( A_1, A_2 \), second node stores \( B_1, B_2 \), third node contains \( A_1 + B_1 \) and \( 2A_2 + B_2 \). Finally, the fourth node stores \( 2A_1 + B_1 \) and \( A_2 + B_2 \). Assume further that the second and fourth nodes fail. To recover both nodes the classical way and simultaneously, they need to contact the remaining alive nodes and download four packets per lost node (4MB) for the regeneration process. However, studies such as [3] and [4] showed that cooperation can help reduce the amount of data exchange to 3MB per node. As can be seen in Fig. 1, an example introduced in [4], the top newcomer downloads \( A_1 \) and \( A_1 + B_1 \) and computes \( B_1 \) for local storage and \( 2A_1 + B_1 \) to transfer to other newcomer. Similarly, the bottom newcomer downloads \( A_2 \) and \( 2A_2 + B_2 \) from alive nodes and compute \( A_2 + B_2 \) for local storage and \( B_2 \) to transfer to other newcomer.

Let us assume a two-layer scenario in which different BSs are involved in the recovery process. We partition packets to half packets (0.5MB) and assume BS communication to be \( w_1 > 1 \) more costly compared to local communication. Also, communication with the satellite/cloud layer costs \( w_2 > 1 \) whereby BS is able to provide at most \( r \) fraction of local downloaded data (\( \beta \)) and satellite/cloud layer stores the entire file. This constraint on the link capacity implies that the number of bytes that can be communicated with higher layer nodes is a linear function of that of local nodes (\( \beta \))\(^1\). In that case, one packet transfer between local nodes and higher layers correspond to \( w_1 \) or \( w_2 \) packet transfers solely between local nodes subject to this limitation.

We provide an example scenario in Fig. 2 in which half packets for \( A_1 \) namely, \( A_1^2 \) and \( A_2^2 \) and for \( A_2 \) namely, \( A_1^2 \) and \( A_2^2 \) are summed before transmission to newcomers. The top newcomer initially resurrects \( B_1^2 + B_2^2 \) and downloads from the other newcomer \( B_2^1 + B_2^2 \). Finally by downloading \( B_2^1 \) from the satellite/cloud layer and \( B_2^2 \) from the BS, it would be able to complete the repair process. Similarly, the bottom newcomer initially computes \( A_2^2 + B_1^2 + A_2^3 + B_2^2 \), and through downloading \( 2A_1^2 + B_1^2 + 2A_2^2 + B_1^2 \) from the other newcomer and the half packets \( 2A_1^2 + B_2^1 \) from the satellite/cloud layer and \( A_2^3 + B_2^2 \) from the BS, respectively, it would be able to complete the repair process. Overall a total bandwidth cost of \( 3 + w_1 + w_2 \) is experienced for the exact repair. In Table I, we compare the bandwidth cost per repaired node using this coding scheme (CoopLayer) based on the example in [4] (CoopLayer) with \( r = 1 \). We have also included no cooperation (NoCoopLocal) and a protocol which only contacts higher layers and download the node content directly from them without local assistance (FullLayer). As can be seen, the coding scheme we present downloads 0.5MB from the BS and 0.5MB from the satellite/cloud layer corresponding

\(^1\) This constraint will be a parameter of our work later on.
to $r = 1$ fraction of the local download ($\beta = 0.5\text{MB}$) and performs best among other schemes except “Centralized”, which is not an option for distributed system design due to single point failure. In this study, we argue that the presented coding scheme uses both layers and is optimal for $r = 1$ for the weight values of Scenario 1 given in Table I. In this case, the cost of CoopLayer is the sum of 1 unit cost from the helper nodes, 0.5 unit cost from the cooperation phase, 0.55 from the BS and 0.85 from the satellite layers. However it turns out that in Scenario 2, using different weight distribution, it is optimal to use only the BS in the repair process. In other words, using $r = 1$ and smaller packetizations, we can show that there exists an optimal coding that can achieve a bandwidth cost of as low as $43/15 \approx 2.867$ (corresponds to $8/3 \approx 2.66$ MBs).

**B. System Model and Information Flow Diagram**

We consider a local cell with $n$ storage nodes and multiple upper layers serving as back-up cluster nodes, each with size $n_l$ with $l = 1, \ldots, M$, such that communicating a symbol with one of the servers at the $l$-th layer are assigned a cost factor $w_l$. Higher layers are assumed to be in possession of the data being repaired at the expense of a higher download cost. A two-layer scenario $M = 2$ is exemplified in Fig. 3 where a node may be receiving data from the satellite/cloud layer, base-stations layer as well as the nodes that belong to the local cellular network layer. For simplicity we shall assume $n_l = 1$ and that clusters are stacked according to non-descending costs, therefore cluster with label 0 would correspond to local cluster and cluster with label $M$ is the cluster that has the highest cost factor. We further assume each link to have limited capacity $b_l\beta$ and hence the number of symbols that can be downloaded from the $l$-th layer BS ($\beta''$) is constrained i.e., $\beta'' \leq b_l\beta$.

Similar to previous analyses, in our study we will resort to information flow graphs to characterize the trade-off region, which are graphical representations of a network in which some information sources are multicast to a set of destination sinks through intermediate nodes. An information flow graph can be described by three types of nodes namely source nodes, intermediate nodes and sink nodes. In general, for the case of single source multiple sink nodes problem, achievable capacity of the network is characterized through max-flow min-cut theorem [13], [2]. In general case of multiple source nodes, achievable capacity is still an open problem. As a pioneering work, Alswede et al. [13] characterized the min-cut max-flow with network coding and proved that linear network codes are sufficient to achieve the min-cut bound in a multicast problem with a single source. In the context of a distributed storage system, a directed acyclic graph called an information flow graph $G$ is used. As mentioned before, $G$ consists of a source $S$, multiple intermediary nodes, and one or more data collectors (DC), which only contact any $k$ nodes for the full reconstruction of the stored file in the network.

We characterize our proposed information flow graph with $(n, k, d, t, \alpha, \gamma, \{b_l\}, \{w_l\})$ tuple using $M$ distinct layers, unlike in [4]. An example graph is illustrated in Fig. 4 for $M = 2$ case. Besides, for a given code parameters $(n, k, d, t, \alpha, \beta, \gamma)$, we denote all possible graphs with $G(n, k, d, t; \alpha, \beta, \gamma, \{\beta''\}_{l=1}^{M})$. Data file $F$ of size $F$ is represented by the source node $S$ at stage $s = 1$. Initial nodes are represented by pairs of vertices $(x_{1n}, x_{O_{out}})$ at stage $s = 0$, where $i$ and $j$ are the time index and device label introduced at time index $i$, respectively. Secondary nodes are represented by tuple $(x_{1n}, x_{Coop}, x_{Coop'}, x_{Out}, x_{Out})$ and data collectors are represented by single node both with stage $s \geq 1$.

In our repair configuration, the initial data repair stage involves contacting $d$ other local live nodes in the same cell and downloading $\beta$ symbols from each through edges $x_{Out}^k \rightarrow x_{1n}$, for $k < i$ and $j \neq j'$. In the next phase, the newcomer contacts $M$ above layers (since $n_l = 1$, we contact a server in each layer) and download $\beta''$ symbols from the $l$-th layer whereby each layer only stores $F_l \geq b_l\beta$ symbols for some $b_l \in \mathbb{R}$ and $j$-th node in $l$-th layer is associated with cost $w_l \geq 1$ per symbol download. In the final phase, all
newcomers undergo a joint local cooperation by downloading \( \beta' \) symbols from at most \( t-1 \) other failed nodes within the same local cell through the edges \( x_{\text{coop},1}^j \rightarrow x_{\text{coop},j'}^k \) for \( j \neq j' \). The newcomer stores \( \alpha \) symbols, and this is represented by a directed edge from \( x_{\text{coop},j'}^k \) to \( x_{\text{out}}^k \) with capacity \( \alpha \). To successfully reconstruct the file, a data collector is connected to \( k \) alive “out” nodes with distinct indices, but not necessarily from the same stage, by \( k \) infinite-capacity edges. We finally realize that most of the previous information flow diagram descriptions would be a special case of this general description.

C. A Fundamental Trade-off

Without loss of generality, we shall assume each layer contains a single base station. Moreover, similar to previous works [14], we introduce auxiliary variables \( r_i \) to be able to express \( \beta'' \) in terms of \( \beta \), i.e., \( \beta'' = r_i \beta \). We characterize the link constraints by bounding the \( r_i \) i.e., \( 0 \leq r_i \leq b_i \) for some fixed \( b_i \in \mathbb{R} \). Based on the regeneration description, the total repair bandwidth cost per failed node (\( \gamma_c \)) is obtained as follows,

\[
\gamma_c(s) = d \beta + (t-1) \beta' + \sum_{i=1}^{M} s_i w_i r_i \beta
\]

(1)

where for some \( \rho \in \{0, 1, \ldots, M\} \) and entries \( s_i \), which defines the number of used BSs with

\[
s_{\rho} = \{ (1, \ldots, 1, 0, \ldots, 0) : \sum_{i=1}^{M} s_i = \rho \}.
\]

(2)

The minimum cut of the flow diagram \( G \) induces the following constraint on the file size

\[
\min_{r_i, u \in P} \left\{ u_0 \alpha + \sum_{j=1}^{g} u_j (d' - \sum_{i=0}^{j-1} u_i) \beta + u_j (t - u_j) \beta' \right\} \geq F
\]

(3)

where \( P = \{ u = (u_i)_{0 \leq i < g} : 1 \leq u_i \leq t \text{ and } \sum_{i=1}^{M} s_i r_i \} \) and \( d' = d + \sum_{i=1}^{M} s_i r_i \). Here, \( u_i \) is the number of nodes contacted in each repair group of size \( t \) during the repair and \( g \) is the number of repairing stages. Note that unlike \( d, d' \) is not necessarily an integer. Therefore, slightly modifying the steps given in [4] we can obtain the file size constraint in (3), which can further be expressed for \( g = 0, \ldots, k \) as,

\[
F \leq \begin{cases} 
\Phi_{g,s_{\rho}} + g \beta (2 - \beta') & \text{if } \beta \geq 2 \beta', \\
\Phi_{g,s_{\rho}} + \psi_{g,t} (\beta' - 2 \beta') & \text{otherwise}
\end{cases}
\]

(4)

where \( \Phi_{g,s} = \alpha (k - g) + g \beta (d' - k + \frac{g}{2}) + \beta g t \) used as a shorthand notation and \( \psi_{g,t} = \frac{gt}{2} + (g - \frac{gt}{2})^2 \).

For a given file size \( F \) and system parameters \( \alpha, d, k, t, w = (w_l)_{1 \leq l \leq M}, b = (b_l)_{1 \leq l \leq M} \), the bandwidth cost-storage trade-off is the solution to the following constrained optimization problem

\[
\min_{s, r_1, \ldots, r_M, \beta, \beta'} \left\{ d \beta + (t-1) \beta' + \sum_{i=1}^{M} s_i w_i r_i \beta \right\}
\]

(5)

subject to the conditional constraint in (4) and

\[
0 \leq \beta, \beta' \leq \frac{F}{d}
\]

(6)

\[
b_i \in \mathbb{R}, r_i \in [0, b_i], \quad l = 1, \ldots, M
\]

(7)

\[
s_{\rho} = [s_1, \ldots, s_M], s_i \in \{0, 1\}
\]

(8)

Due to space constraints, we present a solution of the optimization problem using a few selected parameters in Fig. 5. As can be seen the new trade-off curve, which uses different number of BSs at different operating points, allows better achievable region as compared to two major past works.

III. BS-MSCR AND BS-MBCCR OPERATING POINTS

Two operating points of this trade-off curve are of special importance namely, minimum storage and bandwidth cost operating points.

Theorem 3.1. For a given set values of \( r_1, \ldots, r_M \) in a BS-assisted cooperative repair scenario, the minimum storage and minimum repair bandwidth cost regeneration, namely BS-MSCR and BS-MBCCR points on the tradeoff curve can be characterized as

\[
(\gamma_{\text{BS-MSCR}}, \alpha_{\text{BS-MSCR}}) = \frac{F(d + \sum_{i=1}^{M} s_i w_i r_i + t - 1)}{k(d + \sum_{i=1}^{M} s_i r_i + t - k)}
\]

(9)

\[
(\gamma_{\text{BS-MBCCR}}, \alpha_{\text{BS-MBCCR}}) = \frac{F(2(d + \sum_{i=1}^{M} s_i w_i r_i + t - 1))}{k(2(d + \sum_{i=1}^{M} s_i r_i + t - k))}
\]

(10)

Proof. At the BS-MBCCR operating point, similar to the reasoning given in [4], for a given set of values of \( r_1, \ldots, r_M \), the minimum repair bandwidth cost is the intersection between
\( \beta = 2\beta' \) and \( \beta - \beta' \) planes corresponding to the constraints given in (4) for \( g = k \) and \( \alpha \geq d\beta + (t - 1)\beta' + \sum_{l=1}^{M} s_l r_l \).

To find the intersection, we substitute \( \beta = 2\beta' \) in the \( \beta - \beta' \) plane to get

\[
\alpha(k - k) + k\beta \left( d + \sum_{l=1}^{M} s_l r_l - k + \frac{k + 1}{2} \right) + \frac{k\beta}{2} (t - 1) = F
\]

which subsequently leads to

\[
\beta = \frac{2F}{k\left(2(d + \sum_{l=1}^{M} s_l r_l) + t - k\right)}. \tag{12}
\]

Thus, by substituting \( \beta \) and \( \beta' = \beta/2 \) in the expression of \( \gamma_c(s) \), the result follows. As for the BS-MS-CR operating point, let \( g = lt + q \), where \( l \) is an integer and \( 0 \leq q \leq t \). Note that \( l = 0 \) corresponds to the minimum storage point i.e., \( \alpha = \bar{\gamma} \). Taking into consideration, for a given set of values of \( r_1, \ldots, r_M \), the intersection between \( \beta = \beta' \) with \( \beta - \beta' \) plane subject to the constraint in (4) gives us the BS-MS-CR point.

\[ \square \]

Note that in order to calculate BS-MS-CR and BS-MBCCR operating points, we still need to identify the set of values \( \{r_l\} \). Next theorem demonstrates that minimum bandwidth cost is indeed attained at the upper bounds.

**Theorem 3.2.** For the given values of \( \rho \), \( s_\rho = \{1, \ldots, 1, 0, \ldots, 0\} \) and \( 0 \leq r_l \leq b_l \) for \( l = 1, \ldots, M \), satisfying

\[
\sum_{l=1}^{\rho} w_l r_l \leq \bar{w}_t \sum_{l=1}^{\rho} r_l \tag{13}
\]

with \( \bar{w}_t \in \left\{ \frac{d+t+1}{2(d+t)}, \frac{2d+t+1}{2d+t-k} \right\} \), repair bandwidth cost \( \gamma_c \in \{\gamma_{\text{BS-MS-CR}}, \gamma_{\text{BS-MBCCR}}\} \) is minimized at the upper bounds \( \{b_1, \ldots, b_\rho\} \).

**Proof.** In BS-MBCCR operating point, let us assume that we happen to use \( \rho \) number of base stations associated with the cost factors \( \{w_1, \ldots, w_\rho\} \). In other words, \( \{r_1, \ldots, r_\rho\} \) are non-zero values satisfying the condition

\[
\sum_{l=1}^{\rho} w_l r_l \leq \frac{2d + t - 1}{2d - k + t} \sum_{l=1}^{\rho} r_l. \tag{14}
\]

Note that in order for BSs to reduce the overall bandwidth cost, we would need to satisfy \( \gamma_c(s_\rho) \leq \gamma_c(0) \) which leads to equation (14). Here \( 0 \) denotes the all-zero vector. Accordingly, inequality (14) can be rewritten as follows:

\[
\sum_{l=1}^{\rho} w_l r_l = \tau \bar{w}_t \sum_{l=1}^{\rho} r_l \tag{15}
\]

for some \( \tau \) satisfying \( 0 \leq \tau \leq 1 \). If we reformulate \( \gamma_c(s_\rho) \) in terms of \( \rho \), repair bandwidth cost can be rewritten as,

\[
\gamma_c(s_\rho) = \frac{\bar{w}_t 2d - k + t + 2\tau \sum_{l=1}^{\rho} r_l}{2d - k + t + 2\rho \sum_{l=1}^{\rho} r_l}. \tag{16}
\]

Note that equation (16) is inversely related to \( \sum_{l=1}^{\rho} r_l \) (it can be easily shown by taking the derivative). In other words, in order to minimize \( \gamma_c(s_\rho) \), it is sufficient to maximize the term \( \sum_{l=1}^{\rho} r_l \). A similar proof following the same line of reasoning can be given for BS-MSCR point.

\[ \square \]

The implication of the result of Theorem 3.2 is that we can simply obtain BS-MS-CR and BS-MBCCR points by replacing \( r_l \) with \( b_l \) as,

\[
(\gamma_{\text{BS-MS-CR}}, \rho_{\text{BS-MS-CR}}) = \left( F(d + \sum_{l=1}^{\rho_{\text{BS-MS-CR}}} w_l b_l + t - 1), \frac{F}{k} \right), \tag{17}
\]

\[
(\gamma_{\text{BS-MBCCR}}, \rho_{\text{BS-MBCCR}}) = \left( \frac{F(2(d + \sum_{l=1}^{\rho_{\text{BS-MBCCR}}} w_l b_l) + t - 1)}{k(2d + \sum_{l=1}^{\rho_{\text{BS-MBCCR}}} b_l + t - k)}, \frac{F(2(d + \sum_{l=1}^{\rho_{\text{BS-MBCCR}}} b_l) + t - 1)}{k(2d + \sum_{l=1}^{\rho_{\text{BS-MBCCR}}} b_l + t - k)} \right). \tag{18}
\]

In Algorithm 1, we provide a linear time search algorithm to find \( \rho_{\text{BS-MS-CR}} \) and \( \rho_{\text{BS-MBCCR}} \) i.e., the optimal number of contributing BSs, \( \rho_{\text{min}} \) for both BS-MS-CR and BS-MBCCR operating points. We use \( p_t \) to indicate whether the results corresponds to BS-MS-CR (\( p_t = 1 \)) or BS-MBCCR (\( p_t = 0 \)).

**IV. AN EXPLICIT CONSTRUCTION FOR EXACT BS-ASSISTED MSCR CODE**

Exact repair has many advantages such as reduced maintenance compared to functional repair as indicated in some of the previous works [4]. Exact MSR and MSCR code constructions have been established for a given set of arbitrary

**Algorithm 1 Optimal Number of BSs (\( \rho_{\text{min}} \))**

1: function OptBSNumCal(k, d, t, M, b, w, p_t)
2: \( \rho_{\text{min}} \leftarrow 0 \)
3: for \( i = 1 : M \) do
4: \( \bar{d} \leftarrow d + \sum_{l=1}^{i-1} w_l b_l \), \( \bar{b} \leftarrow d + \sum_{l=1}^{i-1} b_l \) \( \triangleright \) \( b = \{b_l\} \)
5: \( \bar{w}_t \leftarrow p_t \left( \frac{2d + t - 1}{2d + t - k} \right) + (1 - p_t) \left( \frac{2d + r_{i-1}}{2d + t - k} \right) \triangleright p_t \in \{0, 1\} \)
6: if \( w_t > \bar{w}_t \) then
7: \( \triangleright \) break;
8: end if
9: \( \rho_{\text{min}} \leftarrow \rho_{\text{min}} + 1 \)
10: end for
11: return \( \rho_{\text{min}} \)
code parameters in the past [15]. In this section, we provide a family of regeneration codes for BS-assisted cooperative repair with parameters \( \rho \) and \( d = k \leq n - t \). For simplicity, we also assume \( b_1 = 1 \). Similar to previous constructions, we base our construction on multiple maximum distance separable (MDS) codes of length \( n \) with dimension \( k \), consisting of symbols from \( GF(p^q) \) with prime \( p \) and \( n \leq p^q \) for some positive integer \( q \). The code presented in subsection II.A would be a special case of the simplified construction we present in this subsection.

Our construction is based on multiple MDS codes with the same \( k \times n \) generator matrix \( \mathbf{G} = [g_1, g_2, \ldots, g_n] \) where \( g_i \) represents the \( i \)-th column of \( \mathbf{G} \). The file of size \( F \) is divided into \( k(t + \rho) \) chunks where each one of them is considered to be from \( GF(p^q) \). We structure these chunks as a \( (t + \rho) \times k \) massage matrix \( \mathbf{M} \). Thus, we compute \( \mathbf{MG} \) and make the node \( j \) store \( j \)-th column of \( \mathbf{MG} \) for \( j = 1, 2, \ldots, n \). Suppose we represent each row of \( \mathbf{M} \) as \( \mathbf{m}_1^T, \mathbf{m}_2^T, \ldots, \mathbf{m}_{n+\rho}^T \). We have \( k \) message symbols in each row represented as a vector \( \mathbf{m}_i^T = [m_{i1}, m_{i2}, \ldots, m_{ik}] \), each \( m_{ij} \in GF(p^m) \) encoded into the codeword \( \mathbf{m}_i^T \mathbf{G} \). In such a setting, node \( j \) would store \( \mathbf{m}_j^T \mathbf{g}_j \), for \( i = 1, 2, \ldots, t + \rho \).

Suppose that nodes with indices \( j_1, j_2, \ldots, j_t \) fail/depart the cell and become permanently unavailable. In the first phase of the regeneration, the \( t \)-th newcomer indexed by \( j_t \) connects any \( d = k \) remaining nodes, say these nodes are indexed by \( \pi_t(1), \pi_t(2), \ldots, \pi_t(k) \) and downloads \( \mathbf{m}_j^T \mathbf{g}_{\pi_t(1)}, \mathbf{m}_j^T \mathbf{g}_{\pi_t(2)}, \ldots, \mathbf{m}_j^T \mathbf{g}_{\pi_t(k)} \) to successfully compute/reconstruct \( \mathbf{m}_j^T \). In the subsequent cooperative phase, newcomer indexed by \( j_t \) computes \( \mathbf{m}_j^T \mathbf{g}_{j_h} \) to send to the newcomer indexed by \( j_h \), with \( h \neq l \). So a total of \( t - 1 \) chunks are exchanged by each newcomer in this cooperative phase. After the cooperative phase, \( l \)-th newcomer indexed by \( j_l \) has \( t \) chunks namely \( \mathbf{m}_i^T \mathbf{g}_{j_l} \), for \( h = 1, \ldots, t \). In the final phase of the regeneration, we download the remaining \( \rho \) chunks from the available \( \rho \) base stations each providing one chunk of information. We first note that in this case the last two phases of the regeneration are interchangeable. We also note that a chunk of information is of size \( \frac{F}{k(t+\rho)} \) symbols and the total number of chunks downloaded for each newcomer is \( d + \rho + t - 1 \) achieving the minimum bandwidth possible.

V. CONCLUSIONS AND FUTURE WORK

In this work, we have considered the cooperative node repair process of a distributed data storage application in cellular networks while BSs are actively participating in the data regeneration. Since the symbol download between local nodes and BSs may be more costly and limited at times, we have introduced the concept of bandwidth cost and link capacities in our problem formulation. Furthermore, we have characterized appropriate bounds on the file size and solve the resulting optimization problem numerically. In addition, we have found closed form expressions at the minimum storage bandwidth operating points in our setting and conjecture that such expressions can be found for each operating point in the trade-off curve. Our results indicate that a better bandwidth cost-storage space trade-off may be possible in the presence of BSs. We also realize that the cost notion can be applied to storage as well and a natural extension to this work would be to characterize the bandwidth cost-storage cost trade-off. Although we have provided a construction for the exact BS-MSCR operating point, it is valid only for specific parameter selections of the system. We leave different code constructions for general parameters and link capacity settings as a future work.

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